Unbestimmte Integrale

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Aufgaben

Ermitteln Sie die folgenden unbestimmten Integrale.

1. $\int \frac{e^x}{e^x - 3}\ dx$
2. $\int \frac{5}{4x - 3}\ dx$
3. $\int \left(4x^2 \cdot \ln x\right)\ dx$
4. $\int \left(\sqrt{2}x^4 + 7x^2 - 8\right)\ dx$
5. $\int \left(\frac{7}{\sqrt{x}} + \frac{2}{x^3}\right)\ dx$
6. $\int \frac{x^2 + 2x}{3x^5}\ dx$
7. $\int (3x - 1)^4\ dx$
8. $\int \left(5x^2 - 9\right) \cdot (3x - 2)\ dx$
9. $\int \frac{\sin x}{3 - \cos x}\ dx$
10. $\int \frac{2}{3}x^5 - 100x\ dx$
11. $\int (x - 5)^4\ dx$
12. $\int (2x - 5)^4\ dx$
13. $\int -\frac{4}{x^2}\ dx$
14. $\int \frac{1}{(3x + 1)^2}\ dx$
15. $\int \frac{x^2}{(x^3 + 1)^2}\ dx$
16. $\int e^{4x}\ dx$
17. $\int \frac{2x}{(x^2 + 1)^3}\ dx$
18. $\int \frac{x + 2}{(x + 1)^3}\ dx$
19. $\int \frac{2x}{x^2 + 1}\ dx$
20. $\int \sqrt[4]{x^3} + \sqrt[3]{x^5} + \sqrt{x}\ dx$

Lösungen

1.

$$\int \frac{e^x}{e^x - 3}\ dx$$

Substitution:

$$\begin{aligned} e^x - 3 &= t \\ \frac{dt}{dx} &= e^x \\ dx &= \frac{dt}{e^x} \end{aligned}$$

$$\begin{aligned} &= \int \frac{e^x}{t}\ \frac{dt}{e^x} \\ &= \int \frac{1}{t}\ dt \\ &= \ln|t| + C \\ &= \underline{\underline{\ln|e^x - 3| + C}} \end{aligned}$$

2.

$$\int \frac{5}{4x - 3}\ dx$$

Substitution:

$$\begin{aligned} 4x - 3 &= t \\ \frac{dt}{dx} &= 4 \\ dx &= \frac{dt}{4} \end{aligned}$$

$$\begin{aligned} &= \int \frac{5}{t} \cdot \frac{dt}{4} \\ &= \frac{5}{4} \cdot \int \frac{1}{t}\ dt \\ &= \frac{5}{4} \cdot \ln|t| + C \\ &= \underline{\underline{\frac{5}{4} \cdot \ln|4x - 3| + C}} \end{aligned}$$

3.

$$\int \left(4x^2 \cdot \ln x\right)\ dx$$

Partielle Integration:

$$\begin{aligned} u(x) &= \ln x \\ u'(x) &= \frac{1}{x} \\ v'(x) &= 4x^2 \\ v(x) &= \frac{4}{3}x^3 \end{aligned}$$

$$\begin{aligned} &= \frac{4}{3}x^3 \cdot \ln x - \int \frac{4}{3}x^3 \cdot \frac{1}{x}\ dx \\ &= \underline{\underline{\frac{4}{3}x^3 \cdot \ln x - \frac{4}{9}x^3 + C}} \end{aligned}$$

4.

$$
\begin{aligned}
& \int \left(\sqrt{2}x^4 + 7x^2 - 8\right)\ dx \\
= & \int \sqrt{2}x^4\ dx + \int 7x^2\ dx - \int 8\ dx \\
= & \underline{\underline{\frac{\sqrt{2}}{5}x^5 + \frac{7}{3}x^3 - 8x + C}}
\end{aligned}
$$

5.

$$
\begin{aligned}
& \int \left(\frac{7}{\sqrt{x}} + \frac{2}{x^3}\right)\ dx \\
= & \int 7x^{-\frac{1}{2}}\ dx + \int 2x^{-3}\ dx \\
= & 14x^{\frac{1}{2}} - x^{-2} + C \\
= & \underline{\underline{14 \cdot \sqrt{x} - \frac{1}{x^2} + C}}
\end{aligned}
$$

6.

$$
\begin{aligned}
& \int \frac{x^2 + 2x}{3x^5}\ dx \\
= & \int \frac{1}{3x^3}\ dx + \int \frac{2}{3x^4}\ dx \\
= & \int \frac{1}{3}x^{-3}\ dx + \int \frac{2}{3}x^{-4}\ dx \\
= & -\frac{1}{6}x^{-2} - \frac{2}{9}x^{-3} + C \\
= & \underline{\underline{-\frac{1}{6x^2} - \frac{2}{9x^3} + C}}
\end{aligned}
$$

7.

$$
\int (3x-1)^4\ dx
$$

Substitution:

$$
\begin{aligned}
3x - 1 &= t \\
\frac{dt}{dx} &= 3 \\
dx &= \frac{dt}{3}
\end{aligned}
$$

$$
\begin{aligned}
= & \int t^4 \cdot \frac{dt}{3} \\
= & \frac{1}{3} \cdot \frac{1}{5}t^5 + C \\
= & \underline{\underline{\frac{1}{15} \cdot (3x-1)^5 + C}}
\end{aligned}
$$

8.

$$
\begin{aligned}
& \int \left(5x^2-9\right)\cdot(3x-2)\ \ dx \\
=\ & \int \left(15x^3-10x^2-27x+18\right)\ \ dx \\
=\ & \int 15x^3\ dx - \int 10x^2\ dx - \int 27x\ dx + \int 18\ dx \\
=\ & \underline{\underline{\frac{15}{4}x^4-\frac{10}{3}x^3-\frac{27}{2}x^2+18x+C}}
\end{aligned}
$$

9.

$$
\int \frac{\sin x}{3-\cos x}\ dx
$$

Substitution:

$$
\begin{aligned}
3-\cos x &= t \\
\frac{dt}{dx} &= -\sin x \\
dx &= -\frac{dt}{\sin x}
\end{aligned}
$$

$$
\begin{aligned}
=\ & \int \frac{\sin x}{t}\cdot\frac{-dt}{\sin x} \\
=\ & \int -\frac{1}{t}\cdot dt \\
=\ & -\ln|t|+C \\
=\ & \underline{\underline{-\ln|3-\cos x|+C}}
\end{aligned}
$$

10.

$$
\begin{aligned}
& \int \frac{2}{3}x^5-100x\ dx \\
=\ & \frac{2}{3}\cdot\frac{1}{6}x^6-100\cdot\frac{1}{2}x^2+C \\
=\ & \underline{\underline{\frac{1}{9}x^6-50x^2+C}}
\end{aligned}
$$

11.

$$
\int (x-5)^4\ dx
$$

Substitution:

$$
\begin{aligned}
x-5 &= t \\
\frac{dt}{dx} &= 1 \\
dx &= dt
\end{aligned}
$$

$$
\begin{aligned}
=\ & \int t^4\ dt \\
=\ & \frac{1}{5}t^5+C \\
=\ & \underline{\underline{\frac{1}{5}\cdot(x-5)^5+C}}
\end{aligned}
$$

12.

$$\int (2x-5)^4 \ dx$$

Substitution:

$$\begin{aligned} 2x-5 &= t \\ \frac{dt}{dx} &= 2 \\ dx &= \frac{dt}{2} \end{aligned}$$

$$\begin{aligned} &= \int t^4 \cdot \frac{dt}{2} \\ &= \frac{1}{2} \cdot \frac{1}{5} t^5 + C \\ &= \underline{\underline{\frac{1}{10} \cdot (x-5)^5 + C}} \end{aligned}$$

13.

$$\begin{aligned} &\int -\frac{4}{x^2} \ dx \\ &= \int -4x^{-2} \ dx \\ &= 4x^{-1} + C \\ &= \underline{\underline{\frac{4}{x} + C}} \end{aligned}$$

14.

$$\int \frac{1}{(3x+1)^2} \ dx$$

Substitution:

$$\begin{aligned} 3x+1 &= t \\ \frac{dt}{dx} &= 3 \\ dx &= \frac{dt}{3} \end{aligned}$$

$$\begin{aligned} &= \int \frac{1}{t^2} \cdot \frac{dt}{3} \\ &= \frac{1}{3} \cdot \int t^{-2} \ dt \\ &= \frac{1}{3} \cdot \left(-t^{-1}\right) + C \\ &= \underline{\underline{-\frac{1}{3 \cdot (3x+1)} + C}} \end{aligned}$$

15.

$$\int \frac{x^2}{(x^3+1)^2}\ dx$$

Substitution:

$$\begin{aligned} x^3+1 &= t \\ \frac{dt}{dx} &= 3x^2 \\ dx &= \frac{dt}{3x^2} \end{aligned}$$

$$\begin{aligned} &= \int \frac{x^2}{t^2}\cdot\frac{dt}{3x^2} \\ &= \frac{1}{3}\cdot\int t^{-2}\ dt \\ &= \frac{1}{3}\cdot(-1)\cdot t^{-1}+C \\ &= \underline{\underline{-\frac{1}{3\cdot(x^3+1)}+C}} \end{aligned}$$

16.

$$\int e^{4x}\ dx$$

Substitution:

$$\begin{aligned} 4x &= t \\ \frac{dt}{dx} &= 4 \\ dx &= \frac{dt}{4} \end{aligned}$$

$$\begin{aligned} &= \int e^t\cdot\frac{dt}{4} \\ &= \frac{1}{4}\cdot e^t+C \\ &= \underline{\underline{\frac{1}{4}\cdot e^{4x}+C}} \end{aligned}$$

17.

$$\int \frac{2x}{(x^2+1)^3}\ dx$$

Substitution:

$$\begin{aligned} x^2+1 &= t \\ \frac{dt}{dx} &= 2x \\ dx &= \frac{dt}{2x} \end{aligned}$$

$$\begin{aligned} &= \int \frac{2x}{t^3}\cdot\frac{dt}{2x} \\ &= \int t^{-3}\ dt \\ &= -\frac{1}{2}t^{-2}+C \\ &= \underline{\underline{-\frac{1}{2\cdot(x^2+1)^2}+C}} \end{aligned}$$

18.

$$\int \frac{x+2}{(x+1)^3}\ dx$$

$$\begin{aligned}
&= \int \frac{x+2}{t^3}\ dt \\
&= \int \frac{t+1}{t^3}\ dt \\
&= \int \frac{1}{t^2}\ dt + \int \frac{1}{t^3}\ dt \\
&= \int t^{-2}\ dt + \int t^{-3}\ dt \\
&= -t^{-1} - \frac{1}{2}t^{-2} + C \\
&= \underline{\underline{-\frac{1}{x+1} - \frac{1}{2\cdot(x+1)^2} + C}}
\end{aligned}$$

Substitution:

$$\begin{aligned}
x+1 &= t \\
\frac{dt}{dx} &= 1 \\
dx &= dt
\end{aligned}$$

19.

$$\int \frac{2x}{x^2+1}\ dx$$

$$\begin{aligned}
&= \int \frac{2x}{t} \cdot \frac{dt}{2x} \\
&= \int \frac{1}{t}\ dt \\
&= \ln|t| + C \\
&= \underline{\underline{\ln|x^2+1| + C}}
\end{aligned}$$

Substitution:

$$\begin{aligned}
x^2+1 &= t \\
\frac{dt}{dx} &= 2x \\
dx &= \frac{dt}{2x}
\end{aligned}$$

20.

$$\begin{aligned}
&\int \sqrt[4]{x^3} + \sqrt[3]{x^5} + \sqrt{x}\ dx \\
&= \int x^{\frac{3}{4}}\ dx + \int x^{\frac{5}{3}}\ dx + \int x^{\frac{1}{2}}\ dx \\
&= \underline{\underline{\frac{4}{7}x^{\frac{7}{4}} + \frac{3}{8}x^{\frac{8}{3}} + \frac{2}{3}x^{\frac{3}{2}} + C}}
\end{aligned}$$